Introduction to applied optimization

Optimization is one of the most feared topics for calculus students, but it really shouldn't be. Optimization only requires a few simple steps, all of which you already know how to do.

To solve an optimization problem, you'll need to:

1. Write an equation in one variable that represents what you're trying to maximize.
2. Take the derivative, find critical points and draw your wiggle graph.
3. Verify that your solutions are correct based on the real-life situation.

Let's do one of the most common examples.

Example

Given a 5×7 piece of paper with squares cut out from each corner with side-length x,

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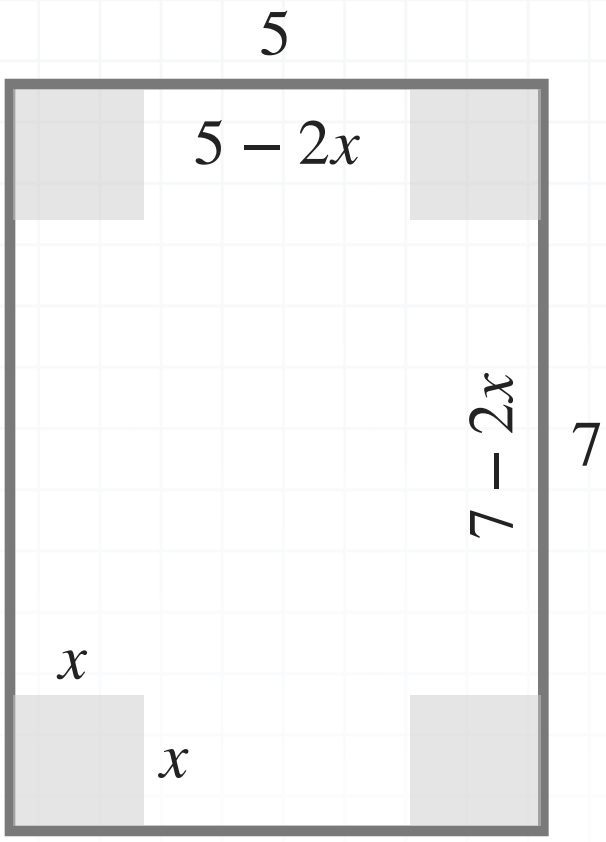

such that folding the sides up will create a box with no top, find the value of x that maximizes the volume of the box.

As soon as you hear volume of a box, you should immediately write down the formula for the volume of a box:

$$V = l \cdot w \cdot h$$

Based on the picture we drew of our problem, we already know our length, width and height, so we rewrite the formula as follows:

$$V = (7 - 2x)(5 - 2x)x$$

Multiplying everything together gives

$$V = \left(35 - 14x - 10x + 4x^2\right) x$$

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$$V = 35x - 14x^2 - 10x^2 + 4x^3$$

$$V = 4x^3 - 24x^2 + 35x$$

Now take the derivative with respect to x.

$$\frac{dV}{dx} = 12x^2 - 48x + 35$$

Find critical points by setting the equation equal to 0 and solving for x.

$$0 = 12x^2 - 48x + 35$$

Using the quadratic formula gives

$$x = \frac{48 \pm \sqrt{2{,}304 - 1{,}680}}{24}$$

$$x = \frac{48 \pm \sqrt{624}}{24}$$

$$x = \frac{12 \pm \sqrt{39}}{6}$$

Our critical points are approximately $x = 3.04$ and $x = 0.96$.

Before we start testing critical points, we should always test our answers for plausibility. Remember: length, width and height can never be negative. If we plug $x = 3.04$ into our length, $(7 - 2x)$, we get a positive number. However, if we plug $x = 3.04$ into our width, $(5 - 2x)$, we get a negative number. Therefore, $x = 3.04$ cannot be a solution to our optimization problem.

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Now we only have to test $x = 0.96$ to make sure it's a local maximum. If it *is* a local max, then 0.96 is the value of x that maximizes the volume of our box.

To test this point, we'll pick a value of x less than 0.96, and another greater than 0.96 and plug them into our derivative.

$$\frac{dV}{dx}(.5) = 12(.5)^2 - 48(.5) + 35$$

$$\frac{dV}{dx}(.5) = 14 > 0$$

$$\frac{dV}{dx}(2) = 12(2)^2 - 48(2) + 35$$

$$\frac{dV}{dx}(2) = -13 < 0$$

Since the derivative is increasing ($14 > 0$) to the left of the critical point, and decreasing ($-13 < 0$) to the right of it, $x = 0.96$ is the value of x that maximizes our volume.

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